TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 173

SIGNIFICANCE OF THE EXPRESSION C_L^3/C_D^2.

By H. von Sanden.

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By H. von Sanden.

The coefficient $\epsilon = C_L^3/C_D^2$ has attained quite an unwarranted prominence through Kann's formula for the ceiling (Technische Berichte, Volume I, No.6, p.231).

$$A_g = 7280 \log 358 \frac{C_L^3/C_D^2 \ \eta^2}{(W/P)^2 \ W/S}$$

One would naturally infer from Kann's formula that an airplane reaches its ceiling with an angle of attack which gives ϵ the maximum value and, in selecting a particular section for the wings of an airplane, stress is laid on the importance of a high maximum value of ϵ, in order to obtain a high ceiling.

Both assumptions are, in general, erroneous, although Kann's formula is correctly derived. P and η, which enter into the formula along with ϵ, are, in fact, not independent of ϵ, although such was assumed to be the case in making the deduction that the maximum value of ϵ gives the maximum altitude. Now a change of ϵ requires, when the weight, engine, propeller and wing area are given, that there should also be a change in the velocity V, affecting, of course, the output of the engine and the efficiency of the propeller.

* From Technische Berichte, Vol. III, No.7, pp. 330-331. (1918)

It is not ϵ, therefore, but $\epsilon\,(P\eta)^2$ that must reach a maximum value.

The velocity at the ceiling and at the maximum climbing speed, lies, generally, below the standard for which the propeller is designed. With this assumption, the torque of the engine may be assumed constant.

According to Bendemann and Madelung (Technische Berichte Vol.II, No.1, p.53), the torque $Q = \mu\,\omega^2\rho\,\pi\,R^4$; the thrust $T = \psi\omega^2\rho\,\pi\,R^4$; the power $P = \mu\,\omega^3\,\rho\,\pi\,R^3$ and the power transmitted to the airplane

$$(P\eta) = \psi\,V\,\omega^2\rho\,\pi\,R^4$$

ψ and μ here being functions of the pitch $\lambda = \frac{V}{DR}$. Let their differential quotients, with respect to λ, be ψ' and μ'

The constancy of the torque makes it a condition that

$$\frac{dQ}{Q} = \frac{\mu'}{\mu}\,d\lambda + 2\,\frac{d\omega}{\omega} = 0.$$

With small changes in V, ω and λ,

$$\frac{d\omega}{\omega} + \frac{d\lambda}{\lambda} - \frac{dV}{V} = 0$$

Hence

$$\frac{d\lambda}{\lambda} = \frac{2\mu}{(2\mu - \lambda\mu')}\,\frac{dV}{V}$$

and from this follows the change in the propeller performance when the speed of flight V, is changed.

$$\frac{d(P\eta)}{P\eta} = \frac{\psi'}{\psi}\,d\lambda + \frac{dV}{V} + 2\,\frac{d\omega}{\omega} =$$

$$= \frac{2\mu(\psi + \lambda\psi') - 3\lambda\psi\mu'}{\psi\,(2\mu - \lambda\mu')}\,\frac{dV}{V}$$

Or, simplified,

$$\frac{d(P\eta)}{P\eta} = \gamma \frac{dV}{V}$$

From $W = C_L V^2 \gamma/2g\ S$, when γ does not change, we obtain

$$\frac{dV}{V} = -\frac{1}{2}\frac{dC_L}{C_L}$$

and, therefore,

$$\frac{d(P\eta)}{P\eta} = -\frac{\gamma}{2}\frac{dC_L}{C_L}$$

In order that $\epsilon\ (P\eta)^2$ and the ceiling may reach a maximum, we must have

$$\frac{3dC_L}{C_L} - 2\frac{dC_D}{C_D} + 2\frac{d(P\eta)}{P\eta} = 0$$

With the above relation between $d(P\eta)$ and dC_L, the condition

$$(3 - \gamma)\frac{dC_L}{C_L} - 2\frac{dC_D}{C_D} = 0$$

Here, γ is a function of λ and, therefore, also of V and, finally, of C_L. With the degree of accuracy here involved, it suffices to put, for λ, the value of γ reached at the ceiling. Since the values of ψ and μ have, hitherto, only been determined for a very few propellers, we must, for the present on the basis of available information, be content to assume a mean value for γ, which may be estimated at 0.5.

The condition can then be established that $\epsilon = \frac{C_L^{2.5}}{C_D^2}$ must be a maximum, a condition which leads to quite practicable angles of attack.

In more exact investigations, it is necessary for us to take into consideration the dependence of the factor Y on the characteristics of the engine and the propeller. The wings and the power plant should not be considered as separate entities, but with regard to their mutual reactions upon each other.

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